

Interference Phenomena in Medium Induced Radiation

J. Casalderrey–Solana,^a E. Iancu,^{a,b}

^aCERN, Theory Division, CH-1211 Geneva, Switzerland

^bInstitut de Physique Théorique, CEA Saclay, F-91191 Gif-sur-Yvette, France

E-mail: jorge.casalderrey@cern.ch, edmond.iancu@cea.fr

Abstract. We consider the interference pattern for the medium-induced gluon radiation produced by a color singlet quark–antiquark antenna embedded in a QCD medium with size L and ‘jet quenching’ parameter \hat{q} . Within the BDMPS–Z regime, we demonstrate that, for a dipole opening angle $\theta_{q\bar{q}} \gg \theta_c \equiv 2/\sqrt{\hat{q}L^3}$, the interference between the medium-induced gluon emissions by the quark and the antiquark is suppressed with respect to the direct emissions. This is so since direct emissions are delocalized throughout the medium and thus yield contributions proportional to L while interference occurs only between emissions at early times, when both sources remain coherent. Thus, for $\theta_{q\bar{q}} \gg \theta_c$, the medium-induced radiation is the sum of the two spectra individually produced by the quark and the antiquark, without coherence effects like angular ordering. For $\theta_{q\bar{q}} \ll \theta_c$, the medium-induced radiation vanishes.

1. Introduction

One of the most spectacular observations of the LHC heavy ion program is the strong modification of jets in a dense QCD medium [1, 2]. Most theoretical descriptions of this modification have focussed on the characterization of the medium-induced gluon radiation off a single parton propagating through the plasma (see [3] for a review). However, due to both jet evolution and in-medium radiation, a jet involves several partons which can radiate simultaneously. It is then necessary to address whether the medium-induced emissions by several sources in such a multi-partonic system can interfere with each other. In vacuum, interference effects are present, frustrating large angle emissions and leading to the *angular ordering* of vacuum showers. The survival of angular ordering for medium-induced radiation has only recently been discussed [4, 5].

To address this problem, we will describe the medium-induced gluon radiation from a $q\bar{q}$ dipole created in the medium. If the dipole angle is sufficiently large, the radiation produced by the q and the \bar{q} has no overlap with each other and thus they are independent. If the dipole opening angle is not that large, and in view of the experience with radiation in the vacuum, one may expect the dipole antenna pattern to be affected by interference effects. However, in the medium this expectation is generally incorrect.

A detailed proof of the of the statement above can be found in [4] and relies on the following properties of the medium-induced radiation by a single source: (i) The formation of a medium-induced gluon takes a time $\tau_f = \sqrt{2\omega/\hat{q}}$. When the gluon is formed it is emitted at a typical angle $\theta_f = (2\hat{q}/\omega^3)^{1/4}$ from the parent quark and it carries a typical transverse momentum $k_f \simeq \omega\theta_f$. (ii) After formation, multiple scattering leads to additional broadening of the gluon spectrum. The final gluon distribution is concentrated within a typical angle $\theta_s = \sqrt{\hat{q}L}/\omega > \theta_f$ around the parent parton. (iii) As long as $\tau_f \ll L$, gluons are emitted all along the medium and the medium-induced gluon spectrum is proportional to $\tau_f L$ (the longitudinal phase-space).

2. Coherence phenomena and interference time scales

The interference phenomena require the two partonic sources to be *coherent* with each other during the gluon formation. In turn, this requirement has *two* aspects:

(i) **Quantum coherence.** The emission process preserves the quantum coherence of the $q\bar{q}$ system so long as the virtual gluon overlaps with both sources during formation. For that to be possible, the transverse resolution of the gluon at the time of formation, as measured by the respective transverse wavelength $\lambda_f = 1/k_f$, should be larger than the typical distance between the q and the \bar{q} . In the vacuum, this condition leads to angular ordering but for medium-induced emissions the situation is more subtle: during formation, the transverse size of the $q\bar{q}$ system increases from $r_{min} \simeq \theta_{q\bar{q}} t_1$ to $r_{max} \simeq \theta_{q\bar{q}} t_2$; here, t_1 and $t_2 = t_1 + \tau_f$ are the times when the emission is initiated and completed respectively. Since the gluon undergoes transverse diffusion during its formation, the relevant size is the *geometric average* of these two scales and the condition of quantum coherence amounts to $\sqrt{r_{min}r_{max}} < \lambda_f$, or $\sqrt{t_1(t_1 + \tau_f)} < \lambda_f/\theta_{q\bar{q}} \equiv \tau_\lambda$ with τ_λ the *transverse resolution time*. It is easy to see that

$$\tau_\lambda = \frac{1}{\theta_{q\bar{q}}(\hat{q}\omega)^{1/4}} = \tau_f \frac{\theta_f}{\theta_{q\bar{q}}}, \quad (1)$$

where the second estimate follows from $k_f \simeq \omega\theta_f$ and $\tau_f \simeq 2/(\omega\theta_f^2)$. This estimate leads to two limiting regimes, depending upon the ratio $\theta_{q\bar{q}}/\theta_f$:

(i.a) For relatively small dipole angles $\theta_{q\bar{q}} \ll \theta_f$, one has $\tau_\lambda \gg \tau_f$ and the definition of τ_λ implies $t_1 < \tau_\lambda$. Since in this regime the BDMPS-Z spectra produced by the two emitters are confined to angles $< \theta_f$ around their respective sources during the formation process, these two spectra overlap but can only interfere over a time τ_λ .

(i.b) For larger dipole angles $\theta_{q\bar{q}} \gg \theta_f$, one has $\tau_\lambda \ll \tau_f$ and therefore $t_1 \ll \tau_f$ as well. The definition of τ_λ leads to $t_1 < \tau_\lambda^2/\tau_f \equiv \tau_{int}$ which introduces a new scale τ_{int} , the *interference time*. This scale can be rewritten as

$$\tau_{int} = \frac{2}{\omega\theta_{q\bar{q}}^2} = \tau_f \left(\frac{\theta_f}{\theta_{q\bar{q}}} \right)^2, \quad (2)$$

where the first expression is recognized as the *vacuum-like* formation time for a gluon emitted at an angle $\sim \theta_{q\bar{q}}$. This is so since, in this regime, interference can only occur for gluons radiated at this large angle.

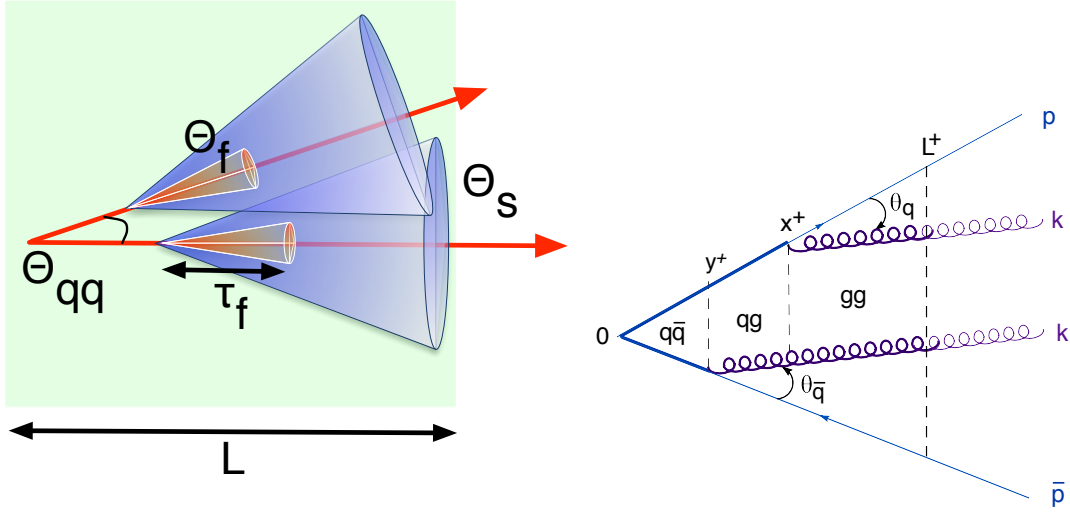


Figure 1. Left: Sketch of the radiation fronts for a relatively large dipole (see text for definition). While the final radiation spectra from each source overlap (blue cone), they are separated at formation (red cone). Right: Feynman graph for interference. The amplitude and complex conjugate amplitude are drawn on top of each other.

(ii) **Color coherence.** In addition to quantum coherence, the existence of interference effects require the preservation of the color coherence between the quark and the antiquark. In the vacuum, the color state of the dipole is conserved until a gluon emission takes place and the interference pattern is governed solely by quantum coherence. In the medium, on the contrary, the interactions with the medium constituents change the color of each of the propagating partons via ‘color rotation’.

For a very energetic parton, this rotation amounts to multiplying the respective wavefunction by a Wilson line. For the $q\bar{q}$ pair we have two such Wilson lines which diverge from each other at constant angle $\theta_{q\bar{q}}$. The color coherence is measured by the 2-point correlation function of these Wilson lines, as obtained after averaging over the fluctuations of the background field. Within the ‘multiple soft scattering approximation’, this 2-point function can be computed and it shows that the quark and the antiquark loose any trace of their original color state after the *decoherence time*

$$\tau_{coh} = \frac{2}{(\hat{q}\theta_{q\bar{q}}^2)^{1/3}} = \tau_f \left(\frac{\theta_f}{\theta_{q\bar{q}}} \right)^{2/3} = \left(\frac{\theta_c}{\theta_{q\bar{q}}} \right)^{2/3} L. \quad (3)$$

3. Classification of dipole sizes.

While the details of the in-medium dipole antenna pattern depend upon all the scales identified above, the phase-space for interference is controlled by the *smallest* of them, $\tau_{min} = \min(\tau_\lambda, \tau_{int}, \tau_{coh})$. As a consequence, the interference contribution to the gluon spectrum, does not scale with the medium length, as the emission from each of the sources does, but with τ_{min} . We can distinguish the following regimes [4]

1. **Very large dipole angles**, $\theta_{q\bar{q}} > \theta_s$. In this case the medium-induced spectra from each source do not overlap and the q and the \bar{q} radiate independently.

2. **Relatively large dipole angles**, $\theta_f < \theta_{q\bar{q}} < \theta_s$. In this regime Eqs. 1, 2 and 3 imply the hierarchy of scales $\tau_{int} < \tau_\lambda < \tau_{coh} < \tau_f$. Accordingly, in this regime, the longitudinal phase-space for interferences is of order $\sim \tau_{int}\tau_f$ and it is suppressed with respect to the corresponding phase-space $\sim \tau_f L$ for direct emissions by a factor

$$\mathcal{R} = \frac{\tau_{int}}{L} \sim \sqrt{\frac{\omega}{\omega_c}} \left(\frac{\theta_f}{\theta_{q\bar{q}}} \right)^2 \ll 1. \quad (4)$$

3. **Relatively small dipole angles** $\theta_c \ll \theta_{q\bar{q}} \ll \theta_f$. In this case, the limitation on the phase-space for interference is due to color coherence, since the ordering of time scales is reverted: $\tau_f \ll \tau_{coh} \ll \tau_\lambda \ll \tau_{int}$. The longitudinal phase-space for interference is now of order $\tau_{coh}\tau_f$ and it is suppressed as compared to the phase-space for direct emissions. Hence the interference contribution is suppressed by

$$\mathcal{R} = \frac{\tau_{coh}}{L} = \left(\frac{\theta_c}{\theta_{q\bar{q}}} \right)^{2/3} \ll 1. \quad (5)$$

Note that, in this case, the medium-induced radiation is distributed at large angles $\theta_q \simeq \theta_{\bar{q}} > \theta_f \gg \theta_{q\bar{q}}$, well outside the dipole cone, and one may wonder why the total radiation is not zero. The reason is that, so long as $\theta_{q\bar{q}} \gg \theta_c$, a $q\bar{q}$ pair immersed in the medium is *not* a ‘color singlet’ anymore, except for a very brief period of time $\sim \tau_{coh}$.

4. **Very small dipoles angles** $\theta_{q\bar{q}} < \theta_c$. For these small angles, the color coherence time τ_{coh} becomes as large as the medium size L , as clear from Eq. 3, and the $q\bar{q}$ pair preserves its color and quantum coherence throughout the medium. Interference effects are not suppressed and they act towards reducing the medium-induced radiation by the dipole. For sufficiently small angles $\theta_{q\bar{q}} \ll \theta_c$, the color decoherence is parametrically small and the total in-medium radiation becomes negligible.

In summary, we have argued that for dipole angles $\theta_{q\bar{q}} \gg \theta_c$, the interference effects for the medium-induced radiation are negligible, so the total BDMPs-Z spectrum by the dipole is the incoherent sum of the spectra produced by the q and the \bar{q} . For smaller angles $\theta_{q\bar{q}} < \theta_c$, the interference effects are not suppressed and they cancel the direct emissions when $\theta_{q\bar{q}} \ll \theta_c$. We observe that for the representative values for $\hat{q} = 10 \text{ GeV}^2/\text{fm}$ and $L = 10 \text{ fm}$, $\theta_c \sim 0.01$ is very small, and most of the dipoles of phenomenological interest will radiate as two independent partonic sources. This simplifies the way towards Monte-Carlo studies of the in-medium jet evolution.

References

- [1] Roland C 2011 *Jet measurements by the CMS experiment in pp and PbPb collisions* (these proceedings).
- [2] Cole C 2011 *ATLAS jet measurements* (these proceedings).
- [3] Casalderrey-Solana J and Salgado C A 2007 Acta Phys. Polon. **B38** 3731-3794. [arXiv:0712.3443].
- [4] Casalderrey-Solana J and Iancu E 2011 [arXiv:1105.1760].
- [5] Mehtar-Tani Y, Salgado C A and Tywoniuk K 2011 Phys. Rev. Lett. **106** 122002. [arXiv:1009.2965].